Int. J. Heat Mass Transfer. Vol. 13, pp. 573–581. Pergamon Press 1970. Printed in Great Britain

LAMINAR HEAT TRANSFER TO A BLUNTED WEDGE*

K. K. CHEN†

Department of the Aerospace and Mechanical Engineering Sciences and Institute for Pure and Applied Physical Sciences, University of California, San Diego, La Jolla, California

(*Received* 25 *November* 1968 *and in revised form* 24 *July* 1969)

Abstract—An analytic solution which describes accurately the energy field in the boundary layer of a blunted wedge with constant wall temperature is provided. For the nose region, the solution is represented by a power series in the streamwise coordinate σ with coefficient functions expressed as linear combinations of universal functions. For flow far from the nose, there is constructed an asymptotic series which contains eigenvalue terms. Since the boundary layer originates as a two-dimensional stagnation-point flow and approaches asymptotically a Falkner–Skan flow with $\beta \neq 1$, the present analytic solution corresponds to a nonsimilar case for both the velocity and energy field which may be suitable for the assessment of the accuracy of numerical and approximate analyses.

NOMENCLATURE

- a, the radius of the blunted nose;
- C, the ratio of density viscosity product, $C \equiv \rho\mu/\rho_e\mu_e$;
- f, modified stream function such that $f_n = u/u_e$;
- $\hat{f}\ldots$, Göertler's universal functions;
- f_n, the coefficient functions of the power series for f;
- F_n, the coefficient functions of the asymptotic series for f;
- g, the ratio of stagnation enthalpy, $g \equiv h_s/h_{s,e}$;
- $\mathfrak{g}$, normalized stagnation enthalpy, $\mathfrak{g} = (g - g_w)/(1 - g_w)$;
- $\hat{g}\ldots$, universal functions for the energy equation;
- g_n, the coefficient functions of the power series for $\mathfrak{g}$;
- G, the ratio of stagnation enthalpy after Euler transformation;
- G_n, the coefficient functions of the power series for G;
- $\hat{G}_n$, the coefficient functions of the asymptotic series for $\mathfrak{g}$;
- h_s, stagnation enthalpy;
- K_n, the constants in front of the eigenvalue terms of the asymptotic series for f;
- M_n, eigenfunction for energy equation;
- N_n, eigenfunction for velocity equation;
- n, normal coordinate;
- s, streamwise coordinate;
- T_n, the constants in front of the eigenvalue terms of the asymptotic series for $\mathfrak{g}$;
- u, velocity component in the streamwise direction;
- v, velocity component in the normal direction;
- V, a constant to normalize the velocity of inviscid flow;
- w, transformed streamwise variable after Euler transformation;
- X, the real part of Z, cf. Fig. 1;
- x, the real part of z, cf. Fig. 1;
- Y, the imaginary part of Z, cf. Fig. 1;
- y, the imaginary part of z, cf. Fig. 1;
- Z, complex variable before conformal mapping, cf. Fig. 1;

* This paper was taken from a part of the author's dissertation submitted to the Faculty of the University of California at San Diego in partial fulfillment of the requirement for the Ph.D. degree (1968).

† Present address: Reentry Technology Directorate, Avco Systems Division, Wilmington, Massachusetts.

z, complex variable after conformal mapping, c.f. Fig. 1.

Greek symbols

β, pressure gradient parameter, $\beta = (2s/u_e)(du_e/ds)$;

β_n, the coefficients of the power series for β;

γ_n, eigenvalues for the energy equation;

η, transformed normal variable, cf. equation (7);

λ_n, eigenvalues for the velocity equation;

μ, viscosity coefficient;

ξ, the imaginary part of ζ, cf. Fig. 1;

ρ, density;

σ, transformed streamwise variable, cf. equation (3);

ζ, intermediate complex variable during conformal mapping, cf. Fig. 1;

$\tilde{\eta}$, the real part of ζ, cf. Fig. 1.

Subscripts

e, refers to the conditions in the external flow;

w, refers to the conditions at the body surface;

0, refers to quantities near the stagnation point;

∞, refers to quantities far from the stagnation point.

1. INTRODUCTION

LAMINAR flows in which the velocity and energy distributions are described by similar solutions can be computed with great accuracy in terms of numerical solutions to a set of ordinary differential equations. More general, nonsimilar flows are described by partial differential equations which may be solved by various methods with various degrees of accuracy. At one end of an accuracy spectrum we have direct numerical integration which in principal is exact and at the other end we have momentum integral methods; series and other methods are intermediate thereto. It would be convenient to have analytic solution of high accuracy to provide gages against which the accuracy of various methods can be judged. This point of view has been exploited for velocity distributions in laminar boundary layers by Van Dyke [1] who provides such a solution for the flow about a parabolic slab and by Chen *et al.* [2], who provide such a solution for the flow about a blunted wedge.

Our purpose here is to present an accurate solution to the energy distribution for the case of the blunted wedge treated in [2]. The velocity distribution is therefore assumed given by [2] and we need present here only those aspects of direct relevance to our calculation of the energy distribution.

The boundary layer on the blunted wedge starts from a two-dimensional stagnation point ($\beta = 1$) and accelerates in the streamwise direction so as to approach a constant $\beta \neq 1$ depending on the wedge angle. Thus the initial and asymptotic solutions for both the velocity and energy distributions are similar. At intermediate stations we are clearly dealing with a nonsimilar flow. The flow may be considered to be either incompressible or compressible but adiabatic with unity Prandtl and Lewis numbers, or more particularly with a nearly constant stagnation enthalpy and composition. Although our solution is for constant wall temperature, the approach may be readily extended to the case of a variable wall temperature.

We carry out two separate calculations. In one we extend the Göertler series method [3] to the solution of the energy equation near the stagnation line. In the second we assume this series solution has been extended sufficiently downstream so that an asymptotic solution based on the eigenfunctions of Chen [4] may be applied to give the approach to the far downstream similar solution.

2. ANALYSIS

Consider a steady laminar flow at a high Reynolds number. For simplicity let the flow be incompressible, passing a semiinfinite, symmetric, blunted wedge with constant wall temperature.

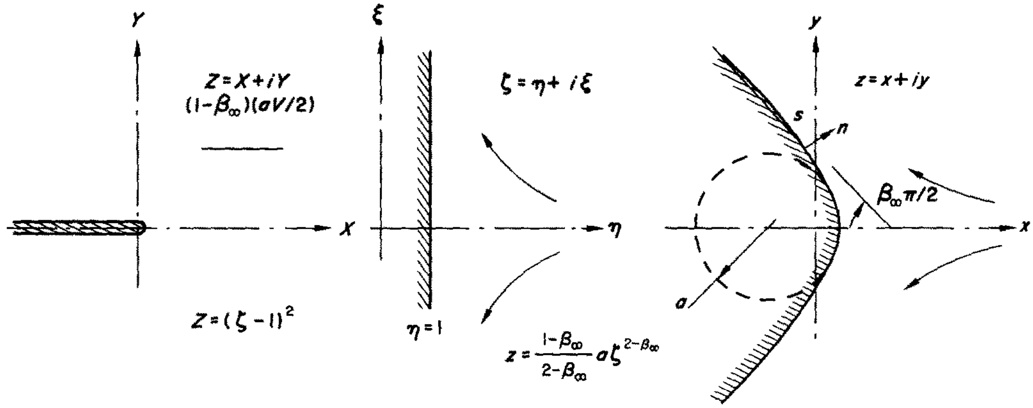

FIG. 1. Generation of the inviscid flow past the blunted wedge by conformal mapping (from [2]).

The inviscid flow

For completeness we review briefly the analysis of [2] providing the description of the inviscid flow which is found by the conformal mappings sketched in Fig. 1. The ζ–z transformation involves the parameter β_∞ which corresponds to the usual Falkner–Skan parameter for wedge flows (cf. Schlichting [5]). For $\beta_\infty = 0$ the wedge degenerates to a parabolic body (cf. Van Dyke [1]), the numerical analysis pertaining to the boundary layer will be carried out below for $\beta_\infty = \frac{1}{2}$, i.e. for a blunted wedge with a 45° half-angle.

The velocity on the surface u_e is found to be given by

$$u_e = V\xi(1 + \xi^2)^{-(1-\beta_\infty)/2}. \tag{1}$$

The distance along the surface from the x-axis is also found to be expressed in terms of ξ in the differential form

$$\frac{\mathrm{d}s}{\mathrm{d}\xi} = (1 - \beta_\infty)\, a(1 + \xi^2)^{(1-\beta_\infty)/2}. \tag{2}$$

For the boundary-layer calculation we shall introduce a transformed streamwise coordinate σ related to s according to

$$\sigma = (Va)^{-1} \int_0^s u_e \,\mathrm{d}s \tag{3}$$

and the usual Falkner–Skan pressure-gradient parameters $\beta \equiv (2\sigma/u_e)(\mathrm{d}u_e/\mathrm{d}\sigma)$ which in these flows varies with σ. Equations (2) and (3) lead to the simple relation between σ and ξ, namely,

$$\sigma = (1 - \beta_\infty)(\xi^2/2), \tag{4}$$

and thus to

$$\beta = \frac{1 + \beta_\infty \xi^2}{1 + \xi^2}. \tag{5}$$

The boundary layer equations

The momentum and energy equation for the boundary layer with the ratio of density viscosity product $C \equiv (\rho\mu/\rho_e\mu_e) = 1$ and the Prandtl number $Pr = 1$ can be written as

$$f_{\eta\eta\eta} + f f_{\eta\eta} + \beta(\sigma)(1 - f_\eta^2) = 2\sigma(f_{\eta\sigma} f_\eta - f_\sigma f_{\eta\eta}) \tag{6}$$

subject to the boundary conditions

$$f(\sigma, 0) = f_\eta(s, 0) = 0$$

$$f_\eta(\sigma, \infty) = 1$$

and

$$\tilde{g}_{\eta\eta} + f\tilde{g}_\eta = 2\sigma(f_\eta \tilde{g}_\sigma - f_\sigma \tilde{g}_\eta) \tag{7}$$

with

$$\tilde{g}(\sigma, 0) = 0, \qquad \tilde{g}(\sigma, \infty) = 1$$

where

$$\eta = \frac{\rho^{\frac{1}{2}} u_e n}{(2\mu V a \sigma)^{\frac{1}{2}}}$$

$$\tilde{g} = (g - g_w)/(1 - g_w),$$

and f_η is the ratio of streamwise velocity components, u/u_e.

The heat transfer rate at wall can be calculated from the value of $(\bar{g}_\eta)_w$ which we shall use as the measure of the accuracy achieved in the subsequent calculation.

*Series solution for nose region**

At the nose of the wedge, say $s = 0$, we have a two-dimensional stagnation point and the velocity and energy fields are described by the similar solutions of f_0 and $\hat{g}_0$ satisfying

$$f_0''' + f_0 f_0'' + (1 - f_0'^2) = 0 \qquad (8)$$

with

$$f_0(0) = f_0'(0) = 0, \qquad f_0'(\infty) = 1$$

and

$$\hat{g}_0'' + f_0 \hat{g}_0' = 0 \qquad (9)$$

with

$$\hat{g}_0(0) = 0, \qquad \hat{g}_0(\infty) = 1.$$

The numerical solution of equation (8) can be found, e.g. in [8] yielding the wall value $f_0''(0) = 1{\cdot}2326$, while the solution of equation (9) can be expressed in an integral form,

$$\hat{g}_0(\eta) = \frac{\int_0^\eta \exp[-\int_0^\eta f_0(\tilde{\eta})\,d\tilde{\eta}]d\eta}{\int_0^\infty \exp[-\int_0^\eta f_0(\tilde{\eta})\,d\tilde{\eta}]\,d\tilde{\eta}} \qquad (10)$$

which gives $\hat{g}_0'(0) = 0{\cdot}57047$.

* A reviewer pointed out that Froessling [6] and Tifford [7] have constructed the universal functions of the temperature field in the physical coordinates s and n, which can be considered as an extension of Blasius series (cf. Schlichting [5]). Though we would expect to be able to convert the numerical values of the universal functions of Blasius series to those of the Göertler series, the labor to do so leads us to prefer our present, direct calculation in σ, η variables. The boundary layer equations in σ and η variables can cover more cases, e.g. axi-symmetrical flow, adiabatic compressible flow and so on.

In the region near the stagnation point, we express β, f and $\bar{g}$ as the series in σ such that

$$\beta \simeq 1 + \beta_1\sigma + \beta_2\sigma^2 + \dots \qquad (11)$$

$$f(s,\eta) \simeq f_0(\eta) + f_1\sigma + f_2\sigma^2 + \dots \qquad (12)$$

$$\bar{g}(s,\eta) \simeq \hat{g}_0(\eta) + g_1\sigma + g_2\sigma^2 + \dots \qquad (13)$$

where β_n's can be obtained from equation (5) for the numerical analysis carried out below, and f_n's can be expressed in terms of the given β_n coefficient and of the Göertler's universal functions $\hat{f}\dots$ (cf. [3]) that

$$\begin{aligned}
f_1 &= \beta_1\hat{f}_1\\
f_2 &= \beta_1^2\hat{f}_{11} + \beta_2\hat{f}_2\\
f_3 &= \beta_1^3\hat{f}_{111} + \beta_1\beta_2\hat{f}_{12} + \beta_3\hat{f}_3\\
f_4\, \beta &\, \beta_1^4\hat{f}_{1111} + \beta_1^2\beta_2\hat{f}_{112} + \beta_1\beta_3\hat{f}_{13} + \beta_2^2\hat{f}_{22} + \beta_4\hat{f}_4 \qquad (14)\\
f_5 &= \beta_1^5\hat{f}_{11111} + \beta_1^3\beta_2\hat{f}_{1112} + \beta_1^2\beta_3\hat{f}_{113} + \beta_1\beta_2^2\hat{f}_{122} + \beta_1\beta_4\hat{f}_{14} + \beta_2\beta_3\hat{f}_{23} + \beta_5\hat{f}_5.
\end{aligned}$$

Substitution of equations (12) and (13) into equation (7) leads to a hierarchy of equations for the g_n functions. If, as an abbreviation, we define the differential operator

$$L_k(g) = g'' + f_0 g' - 2kf_0' g,$$

we have the following equations for the functions g_n,

$$\begin{aligned}
L_1(g_1) &= -3f_1\hat{g}_0'\\
L_2(g_2) &= -3f_1 g_1' - 5f_2\hat{g}_0' + 2f_1' g_1\\
L_3(g_3) &= -3f_1 g_2' - 5f_2 g_1' - 7f_3\hat{g}_0' + 4f_1' g_2 + 2f_2' g_1\\
L_4(g_4) &= -3f_1 g_3' - 5f_2 g_2' - 7f_3 g_1' - 9f_4\hat{g}_0' + 6f_1' g_3 + 4f_2' g_2 + 2f_3' g_1 \qquad (15)\\
L_5(g_5) &= -3f_1 g_4' - 5f_2 g_3' - 7f_3 g_2' - 9f_4 g_1' - 11f_5\hat{g}_0' + 8f_1' g_4 + 6f_2' g_3 + 4f_3' g_2 + 2f_4' g_1.
\end{aligned}$$

Following the same approach as Göertler [3], we express the g_n functions in terms of a series of universal functions which are defined as follows

$$
\begin{aligned}
g_1 &= \beta_1 \hat{g}_1 \\
g_2 &= \beta_1^2 \hat{g}_{11} + \beta_2 \hat{g}_2 \\
g_3 &= \beta_1^3 \hat{g}_{111} + \beta_1 \beta_2 \hat{g}_{12} + \beta_3 \hat{g}_3 \\
g_4 &= \beta_1^4 \hat{g}_{1111} + \beta_1^2 \beta_2 \hat{g}_{112} + \beta_1 \beta_3 \hat{g}_{13} \\
&\quad + \beta_2^2 \hat{g}_{22} + \beta_4 \hat{g}_4 \\
g_5 &= \beta_1^5 \hat{g}_{11111} + \beta_1^3 \beta_2 \hat{g}_{1112} \\
&\quad + \beta_1^2 \beta_3 \hat{g}_{1113} + \beta_1 \beta_2^2 \hat{g}_{122} \\
&\quad + \beta_1 \beta_4 \hat{g}_{14} + \beta_2 \beta_3 \hat{g}_{23} + \beta_5 \hat{g}_5 .
\end{aligned}
\tag{16}
$$

Substitution of (14) and (16) into (15) leads to the following differential equations for the functions $\hat{g}\ldots(\eta)$.

We have the same boundary conditions for all $\hat{g}\ldots$, namely

$$\hat{g}\ldots(0) = \hat{g}\ldots(\infty) = 0.$$

The numerical technique used to solve equations in (17) is essentially the same as that used by Göertler [3]. A brief outline is given here. Let

$$\hat{g}\ldots = C\hat{g}\ldots_c(\eta) + \hat{g}\ldots_p(\eta),$$

where $\hat{g}\ldots_c$ satisfies

$$L_k(\hat{g}\ldots_c) = 0,$$

$$\hat{g}\ldots_c(0) = 0, \qquad \hat{g}\ldots_c'(0) = 1;$$

$\hat{g}\ldots_p$ satisfies

$$L_k(\hat{g}\ldots_p) = R\ldots$$

$$\hat{g}\ldots_p(0) = \hat{g}\ldots_p'(0) = 0$$

$$
\begin{aligned}
k = 1:\ & L_1(\hat{g}_1) = -3\hat{f}_1\hat{g}_0' \\
k = 2:\ & L_2(\hat{g}_{11}) = -3\hat{f}_1\hat{g}_1' - 5\hat{f}_{11}\hat{g}_0' + 2\hat{f}_1'\hat{g}_1 \\
& L_2(\hat{g}_2) = -5\hat{f}_2\hat{g}_0' \\
k = 3:\ & L_3(\hat{g}_{111}) = -3\hat{f}_1\hat{g}_{11}' - 5\hat{f}_{11}\hat{g}_1' - 7\hat{f}_{111}\hat{g}_0' + 4\hat{f}_1'\hat{g}_{11} + 2\hat{f}_{11}'\hat{g}_1 \\
& L_3(\hat{g}_{12}) = -{}^3\hat{f}_1\hat{g}_2' - 5\hat{f}_2\hat{g}_1' - 7\hat{f}_{12}\hat{g}_0' + 4\hat{f}_1'\hat{g}_2 + 2\hat{f}_2'\hat{g}_1 \\
& L_3(\hat{g}_3) = -7\hat{f}_3\hat{g}_0' \\
k = 4:\ & L_4(\hat{g}_{1111}) = -3\hat{f}_1\hat{g}_{111}' - 5\hat{f}_{11}\hat{g}_{11}' - 7\hat{f}_{111}\hat{g}_1' - 9\hat{f}_{1111}\hat{g}_0' + 6\hat{f}_1'\hat{g}_{111} + 4\hat{f}_{11}'\hat{g}_{11} + 2\hat{f}_{111}'\hat{g}_1 \\
& L_4(\hat{g}_{112}) = -3\hat{f}_1\hat{g}_{12}' - 5\hat{f}_{11}\hat{g}_2' - 5\hat{f}_2\hat{g}_{11}' - 7\hat{f}_{12}\hat{g}_1' - 9\hat{f}_{112}\hat{g}_0' + 6\hat{f}_1'\hat{g}_{12} + 4\hat{f}_{11}'\hat{g}_2 \\
& \qquad + 4\hat{f}_2'\hat{g}_{11} + 2\hat{f}_{12}\hat{g}_1 \\
& L_4(\hat{g}_{13}) = -3\hat{f}_1\hat{g}_3' - 7\hat{f}_3\hat{g}_1' - 9\hat{f}_{13}\hat{g}_0' + 6\hat{f}_1'\hat{g}_3 + 2\hat{f}_3'\hat{g}_1 \\
& L_4(\hat{g}_{22}) = -5\hat{f}_2\hat{g}_2' - 9\hat{f}_{22}\hat{g}_0' + 4\hat{f}_2'\hat{g}_2 \\
& L_4(\hat{g}_4) = -9\hat{f}_4\hat{g}_0' \\
k = 5:\ & L_5(\hat{g}_{11111}) = -3\hat{f}_1\hat{g}_{1111}' - 5\hat{f}_{11}\hat{g}_{111}' - 7\hat{f}_{111}\hat{g}_{11} - 9\hat{f}_{1111}\hat{g}_1' - 11\hat{f}_{11111}\hat{g}_0' + 8\hat{f}_1'\hat{g}_{1111} \\
& \qquad + 6\hat{f}_{11}'\hat{g}_{111} + 4\hat{f}_{111}'\hat{g}_{11} + 2\hat{f}_{4111}\hat{g}_1 \\
& L_5(\hat{g}_{1112}) = -3\hat{f}_1\hat{g}_{112}' - 5\hat{f}_{11}g_{12}' - 5\hat{f}_2g_{111}' - 7f_{12}g_{11}' - 7\hat{f}_{111}\hat{g}_2' - 9\hat{f}_{112}\hat{g}_1' \\
& \qquad - 11\hat{f}_{1112}\hat{g}_0' + 8\hat{f}_1'\hat{g}_{112} + 6\hat{f}_{11}'\hat{g}_{12} + 6\hat{f}_2'\hat{g}_{111} + 4\hat{f}_{111}'\hat{g}_2 + 2\hat{f}_{112}\hat{g}_1 \\
& L_5(\hat{g}_{113}) = -3\hat{f}_1\hat{g}_{13}' - 5\hat{f}_{11}\hat{g}_3' - 7\hat{f}_3\hat{g}_{11}' - 9\hat{f}_{13}\hat{g}_1' - 11\hat{f}_{113}\hat{g}_0' + 8\hat{f}_1'\hat{g}_{13} + 6\hat{f}_{11}'\hat{g}_3 \\
& \qquad + 4\hat{f}_3'\hat{g}_{11} + 2\hat{f}_{13}'\hat{g}_1 \\
& L_5(g_{122}) = -3\hat{f}_1\hat{g}_{22}' - 5\hat{f}_2\hat{g}_{12}' - 7\hat{f}_{12}\hat{g}_2' - 9\hat{f}_{22}\hat{g}_1' - 11\hat{f}_{122}\hat{g}_0' + 8\hat{f}_1'g_{22} + 6\hat{f}_2'\hat{g}_{12} \\
& \qquad + 4\hat{f}_{12}'\hat{g}_2 + 2\hat{f}_{22}'\hat{g}_1 \\
& L_5(\hat{g}_{14}) = -3\hat{f}_1\hat{g}_4' - 9\hat{f}_4\hat{g}_1' - 11\hat{f}_{14}\hat{g}_0' + 8\hat{f}_1'\hat{g}_4 + 2\hat{f}_4'\hat{g}_1 \\
& L_5(\hat{g}_{23}) = -5\hat{f}_2\hat{g}_3' - 7\hat{f}_3\hat{g}_2' - 11\hat{f}_{23}\hat{g}_0' + 6\hat{f}_2'\hat{g}_3 + 4\hat{f}_3'\hat{g}_2 \\
& L_5(\hat{g}_5) = -11\hat{f}_5\hat{g}_0'.
\end{aligned}
\tag{17}
$$

with $R\ldots$, the forcing function of the corresponding equation in (17); and C is a constant to be determined from the boundary condition that

$$0 = \hat{g}\ldots(\infty) = C\hat{g}\ldots_c(\infty) + \hat{g}\ldots_p(\infty).$$

Extensive tables of these $\hat{g}\ldots(\eta)$ functions for g_n, where $n = 1, 2, 3, 4$ and 5, were calculated and are available upon request from the author. In the present paper, we list only the numerical values of $\hat{g}'\ldots(0)$ in Table 1.

Table 1. The numerical values of $\hat{g}\ldots'(0)$

....	$\hat{g}\ldots'(0)$	....	$\hat{g}\ldots'(0)$
0	0·57047	5	0·06030
1	0·06219	23	−0·02600
2	0·06354	14	−0·02721
11	−0·01669	122	0·01319
3	0·06283	113	0·01360
12	−0·03083	1112	−0·00767
111	0·00550	11111	0·00096
4	0·06162		
22	−0·01398		
13	−0·02884		
112	0·01477		
1111	−0·00215		

Since from our wedge flow the coefficients β_n are known from (5), we may compute the first 5 g_n functions from (16) and from (13) the stagnation enthalpy in the boundary layer to some finite σ, less than the radius of convergence of the series in σ and determined by the selected accuracy requirements on some boundary layer property, e.g. on $(g_n)_w$.

Extending the series solution

In order to extend the radius of convergence, as well as to increase the accuracy of the series, we recast the series in (13) by means of the Euler transformation. Introducing $w = \sigma/(q + \sigma)$ so that $\tilde{g}(\sigma, \eta)$ becomes $G(w, \eta)$ where

$$G(w, \eta) = \hat{g}_0(\eta) + \sum_{n=1} w^n G_n(\eta) \tag{18}$$

$$\begin{aligned} G_1 &= qg_1 \\ G_2 &= qg_1 + q^2 g_2 \\ G_3 &= qg_1 + 2q^2 + q^3 g_3 \\ G_4 &= qg_1 + 3q^2 g_2 + 3q^3 g_3 + q^4 g_4 \\ G_5 &= qg_1 + 4q^2 g_2 + 6q^3 g_3 + 4q^4 g_4 \\ &\quad + q^5 g_5. \end{aligned} \tag{19}$$

The determination of the value of q is ambiguous: e.g. it may be selected in such a way that the series in (18) converges as fast as possible, e.g. [9]. It may also be determined on the basis of an estimation of the radius of convergence of the series. With q selected we may in some cases employ the technique of Shanks [10] for accelerating the convergence of a sequence of partial sums.

Asymptotic series for the stagnation enthalpy far downstream

In order to examine how the stagnation enthalpy will approach a similar solution as $\sigma \to \infty$, asymptotic expansions to the solutions of equations (6) and (7) with respect to large σ are employed. The asymptotic expansion to equation (1) has been presented in [2] as

$$\begin{aligned} f(s, \eta) \sim\ & f_\infty(\eta) + F_1(\eta)\sigma^{-1} + K_1 N_1(\eta)\sigma^{-\lambda_1/2} \\ & + F_2(\eta)\sigma^{-2} + K_2 N_2(\eta)\sigma^{-\lambda_2/2} \\ & + \tilde{G}_{11}(\eta)\sigma^{-(1+\lambda_1/2)} + F_3(\eta)\sigma^{-3} + \ldots \end{aligned} \tag{20}$$

where f_∞ is the solution of Falkner–Skan equation with $\beta_\infty \equiv \beta(\infty)$. In a similar manner we write the asymptotic expansion to equation (7) as

$$\begin{aligned} \tilde{g}(s, \eta) \sim\ & g_\infty + T_1 M_1 \sigma^{-\gamma_1/2} + \hat{G}^1(\eta)\sigma^{-1} \\ & + C_1 \hat{G}_{\lambda_1/2}\sigma^{-\lambda_1/2} + T_1 \hat{G}_{(\lambda_1/2)+1}\sigma^{-(1+\gamma_1/2)} \\ & + T_2 M_2 \sigma^{-\gamma_2/2} + \hat{G}_2(\eta)\sigma^{-2} + \ldots. \end{aligned} \tag{21}$$

Substitution of equations (20) and (21) into equation (7) and collection of like powers of

σ yield the sequence of equations

$$g''_\infty + f_\infty g'_\infty = 0$$

with $g_\infty(0) = 0, g_\infty(\infty) = 1$;

$$M''_1 + f_\infty M'_1 + \gamma_1 f'_\infty M_1 = 0$$

with $M_1(0) = M_1(\infty) = 0$;

$$\hat{G}''_1 + f_\infty \hat{G}_1 = F_1 g'_\infty$$

with $\hat{G}_1(0) = \hat{G}_1(\infty) = 0$;

$$\tilde{G}''_{\lambda_1/2} + f_\infty \hat{G}'_{\lambda_1/2} + \lambda_1 f'_\infty \hat{G}_{\lambda_1/2} = (\lambda_1 - 1) N_1 g'_\infty$$

with $\hat{G}_{\lambda_1/2}(0) = \hat{G}_{\lambda_1/2}(\infty) = 0.$ (22)

The first function g_∞ can be expressed in an integral form as in (10) with the substitution of f_∞ for f_0. The second function M_1 is the first eigenfunction in [4] with $f = f_\infty$. Substitution of $F_1 = (\frac{1}{2})(1 - \beta_\infty)^2 . (\eta f'_\infty - f_\infty)$, as it is indicated in [2] into (22) leads to an exact solution for $\hat{G}_1$, i.e. $\hat{G}_1 = \frac{1}{4}(1 - \beta_\infty)^2 \eta g'_\infty$. As to the fourth function $\hat{G}_{\lambda_1/2}$, we cannot find a general exact solution except for $\beta_\infty = 0$. In this special case, we have $\lambda_1 = 0$ and $N_1(\eta) = (\eta f'_\infty - f_\infty)/f''_\infty(0)$ so that $\hat{G}_{\lambda_1/2} = \eta g'_\infty / f''_\infty(0)$. However we can find solutions for other functions in (21) numerically in a straightforward manner. The constants $T_1, T_2, \ldots$, in (21) depend upon the details of the flow near the nose as discussed below.

3. NUMERICAL RESULTS FOR $\beta_\infty = \frac{1}{2}$

Consider a blunted wedge of 45° half-angle, i.e. $\beta_\infty = \frac{1}{2}$. From the substitution of numerical results in Table 1 to equations (16) and (13), we find

$$\tilde{g}_\eta(s, \eta) = 0.57047 - 0.12438\sigma + 0.44159\sigma^2 - 1.56145\sigma^3 + 5.58451\sigma^4 - 20.21776\sigma^5 \ldots. \quad (23)$$

The next step is to employ the Euler transformation $w = \sigma/(q + \sigma)$ in order to increase the radius of convergence of (23). The appropriate value of the constant q depends on that radius which can be assumed to be or expected to be the same with that of velocity filed in [2], i.e. $q = \frac{1}{4}$. In terms of w, equation (23) becomes

$$G_\eta(w, 0) = 0.57047 - 0.03110w - 0.00350w^2 - 0.00029w^3 + 0.00033w^4 + 0.00043w^5. \quad (23a)$$

With $w = 1$, which corresponds to $\sigma \to \infty$, equation (23a) gives the sequence of partial sums of

0.5394 0.5359 0.5356 0.5359 0.5363,

compared with the exact similar solution for $\sigma \to \infty$, $g'_\infty(0) = 0.5390$ [4]. We thus see that the Euler transformation applied to five terms of the series solution yields the asymptotic value of $g'_\infty(0)$ within 0.5 per cent.

In order to determine the nature of approach of $g_\eta(\sigma, 0)$ to its asymptotic value, we find the numerical value of its first 3 terms of (21), i.e.

$$\tilde{g}_\eta(s, 0) \sim 0.5390 + T_1 \sigma^{-0.8261} + 0.06737\sigma^{-1} + 0(\sigma^{-1.546}). \quad (24)$$

The numerical value of T_1 can be approximately estimated by the consideration of the power series (23a) for $g_\eta(\sigma, 0)$. The procedure in outline is as follows: equation (24) can be rewritten in terms of $(1 - w)$ as

$$(\tilde{g}_\eta)_w \sim 0.5390 + 5^{0.8261} T_1 (1 - w)^{0.8261} + 0.2695(1 - w) + 0[(1 - w)^{1.546}]. \quad (25)$$

Subtraction of equation (25) from equation (23a) leads to

$$5^{0.8261}(-T_1)(1 - w)^{0.8261} = 0.2380 - 0.2384w + 0.00350w^2 + 0.00029w^3 + 0.00033w^4 + 0.00043w^5 + 0.00042w^4 + 0.00010w^5.$$

Factoring 0.2380 from the right-hand side and raising both sides to $(0.8261)^{-1}$ power, we obtain

$$5(-T_1)^{1.2105}(1 - w) = (0.2380)^{1.2105}(1 - 1.2125w + 0.1456w^2 + 0.0314w^3 + 0.0117w^4 + 0.0056w^5 + \ldots).$$

We once again rewrite the right-hand side in powers of $(1 - w)$ and find

$$5(-T_1)^{1\cdot 2105}(1 - w)$$
$$= (0\cdot 2380)^{1\cdot 2105}[(1 - 1\cdot 2125 + 0\cdot 1456 + 0\cdot 0314 + 0\cdot 0117 + 0\cdot 0056 + \ldots) + (1 - w)(1\cdot 2125 - 0\cdot 2912 - 0\cdot 0943 - 0\cdot 0467 - 0\cdot 0282 - \ldots) + 0(1 - w)^2].$$

The terms in the first 2 coefficients decrease faster than $1/n^2$. Applying the summation formula of Shanks [10] to these two coefficients, we obtain $-0\cdot 01$ and $0\cdot 64$. This numerical results seems to agree with our expectation that the first coefficient indeed converges to zero. Therefore, the second coefficient can be employed to estimate T_1, i.e.

$$T_1 = -0\cdot 2380 \ \frac{0\cdot 64}{5}^{0\cdot 8261} = -0\cdot 044.$$

We thus conclude that the heat transfer rate far downstream is given by

$$(\mathcal{G}_\eta)_w \sim 0\cdot 5390 - \sigma^{-0\cdot 8261} + 0\cdot 0674\sigma^{-1} + 0\ (\sigma^{-1\cdot 546}).$$

We thus see that the series solution developed for the nose region, extended by means of the Euler transformation, and the asymptotic solution far downstream from the nose provide a gage for the assessment of the accuracy of numerical and/or approximate solutions of the energy field on a blunted wedge and complement the corresponding solutions for the velocity field given in [2].

ACKNOWLEDGEMENTS

The author is pleased to acknowledge Professor Paul A. Libby for suggesting this problem and to thank him for his continuous guidance and help throughout this study. This research was supported by the Advanced Research Projects Agency (Project DEFENDER) under Contract No. DA-31-124-ARO-D-257, monitored by the U.S. Army Research Office, Durham.

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TRANSPORT DE CHALEUR LAMINAIRE À UN DIÈDRE ARRONDI

Résumé—Une solution analytique est fournie, qui décrit avec précision le champ d'énergie dans la couche limite d'un dièdre arrondi avec une température pariétale constante. Pour la région du nez, la solution est représentée par une série en puissances de la coordonnée le long de l'écoulement σ, avec des coefficients variables exprimés comme des combinaisons linéaires de fonctions universelles. Pour l'écoulement loin du nez, on a construit une série asymptotique qui contient des valeurs propres. Puisque la couche limite commence comme un écoulement autour d'un point d'arrêt bidimensionnel et approche asymptotiquement d'un écoulement de Falkner-Skan avec $\beta \neq 1$, la solution analytique actuelle correspond à un cas en non-similitude à la fois pour le champ de vitesse et le champ d'énergie qui peut être convenable pour la fixation de la précision des analyses numériques et approchées.

LAMINARE WÄRMEÜBERTRAGUNG AN EINEN ABGESTUMPFEN KEIL.

Zusammenfassung— Es wird eine analytische Lösung gebracht, die exakt das Energiefeld in der Grenzschicht eines abgestumpften Keils bei konstanter Wandtemperatur beschreibt. Für das Gebiet der Nase wird die Lösung durch eine Potenzreihe mit der Strömungskoordinate σ und Koeffizientenfunktionen dargestellt. die als Linearkombinationen universeller Funktionen ausgedrückt werden. Für die Strömung in grossem Abstand von der Nase wird eine asymptotische Reihe entwickelt, die Eigenwertbeziehungen enthält. Da die Grenzschicht mit einer zweidimensionalen Staupunktsströmung beginnt und sich asymptotisch einer Falkner-Skan-Strömung mit $\beta = 1$ nähert, entspricht die hier dargestellte analytische Lösung einem nichtähnlichen Fall sowohl für das Geschwindigkeits- als auch für das Temperaturfeld, was für die Abschätzung der Genauigkeit numerischer und augenäherter Lösungen verwendet werden kann.

ЛАМИНАРНАЯ ПЕРЕДАЧА ТЕПЛА К ЗАТУПЛЕННОМУ КЛИНУ

Аннотация—Приводится решение в аналитической форме, точно описывающее поле энергии в пограничном слое затупленного клина с постоянной температурой стенок. В области носовой части, решение выражено в виде ряда по степеням координата, отсчитываемая в направлении потока с функциями коэффициента, как линейные комбинации универсальных функций. Для потока вдали от носовой части построен асимптотический ряд, содержащий термы собственного значения. Так как пограничный слой возникает в виде плоского течения вблизи критической точки и приближается асимптотический к потоку Фоулкнер-Скана с $\beta \neq 1$, настоящее аналитическое решение соответствует неподобному случаю как для поля скорости, так и для поля энергии, что может подойти для оценки точности числового и приближенного анализов.